The equation of state in Standard Model

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The tree level Lagrangian. The phenomenological basis for the formulation of the Standard Model (MS) is given by the following empirical facts:
The tree level Lagrangian. The phenomenological basis for the formulation of the Standard Model (MS) is given by the following empirical facts:

- The $SU(2) \times U(1)$ family structure of the fermions: The fermions appear as families with left-handed doublets and right-handed singlets:

\[
\begin{align*}
&\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \\
&\begin{pmatrix} e_R \\ \mu_R \end{pmatrix}, \begin{pmatrix} \tau_R \end{pmatrix}, \begin{pmatrix} u_R \end{pmatrix}, \begin{pmatrix} d_R \end{pmatrix}, \begin{pmatrix} c_R \end{pmatrix}, \begin{pmatrix} s_R \end{pmatrix}, \begin{pmatrix} t_R \end{pmatrix}, \begin{pmatrix} b_R \end{pmatrix}
\end{align*}
\]
Introduction.

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- They can be characterized by the quantum numbers of weak isospin $I, I_3$ and the weak hypercharge $Y$.
- Between the quantum numbers classifying the fermions with respect to the group $SU(2) \times U(1)$ and their electric charges $Q$ the Gell-Mann-Nishijima relation is valid.

$$Q = I_3 + \frac{Y}{2}$$
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$$Q = I_3 + \frac{Y}{2}$$

- The existence of vector bosons: $\gamma, W^+, W^-, Z$. 
Introduction.

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- This empirical structure can be embedded in a gauge invariant field theory of the unified electromagnetic and weak interactions by interpreting $SU(2) \times U(1)$ as the group of gauge transformations under which the Lagrangian is invariant.

- This full symmetry has to be broken by the Higgs mechanism down to the electromagnetic gauge symmetry; otherwise the $W^\pm$, $Z$ bosons would also be massless.

- The Standard Model requires a single scalar field (Higgs field) which is a doublet under $SU(2)$. 

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The classical Lagrangian.

The **electroweak Lagrangian** is given in the form:

\[ \mathcal{L} = \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_{\text{Yukawa}} \]
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- **Gauge fields.**
  - \( SU(2) \times U(1) \) is a non-Abelian group which is generated by the isospin operators \( I_1, I_2, I_3 \) and the hypercharge \( Y \).
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- **Gauge fields.**
  - \( SU(2) \times U(1) \) is a non-Abelian group which is generated by the isospin operators \( I_1, I_2, I_3 \) and the hypercharge \( Y \).
  - Each of these charges is associated with a vector field: a isotriplet of vector fields \( W_{\mu}^{1,2,3} \) with \( I^{1,2,3} \) and a isosinglet field \( B_{\mu} \) with \( Y \).
The classical Lagrangian.

- The field strength tensors:

\[ W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \varepsilon_{abc} W^b_\mu W^c_\nu \]

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]
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- The gauge field Lagrangian:

\[ \mathcal{L}_G = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \]
The classical Lagrangian.

- Fermion fields and fermion-gauge interaction.
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- Lagrangian $\mathcal{L}_F$:

$$\mathcal{L}_F = \sum \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \sum \bar{\psi}_R i \gamma^\mu D_\mu \psi_R$$
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- The covariant derivative

$$D_\mu = \partial_\mu - ig_2 I_a W^a_\mu + ig_1 \frac{Y}{2} B_\mu$$
The classical Lagrangian.

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For singlets $\Rightarrow I_a = 0.$
The classical Lagrangian.

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- A single complex scalar doublet field

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The classical Lagrangian.

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\[ \Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \]

- Lagrangian \( \mathcal{L}_H \)

\[ \mathcal{L}_H = (D_\mu \Phi)^+(D^\mu \Phi) - V(\Phi) \]
The classical Lagrangian.

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- Lagrangian \( \mathcal{L}_H \)

\[ \mathcal{L}_H = (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi) \]

with the covariant derivative

\[ D_\mu = \partial_\mu - ig_2 I_a W_\mu^a + i \frac{g_1}{2} B_\mu \]
The classical Lagrangian.

- Potential $V(\Phi)$:

$$V(\Phi) = -\mu^2 \Phi^{+} \Phi + \frac{\lambda}{4} (\Phi^{+} \Phi)^2$$

where $v = \frac{2\mu}{\sqrt{\lambda}}$. 
The classical Lagrangian.

- **Potential** $V(\Phi)$:

  $$V(\Phi) = -\mu^2 \Phi^+ \Phi + \frac{\lambda}{4}(\Phi^+ \Phi)^2$$

  where $\nu = \frac{2\mu}{\sqrt{\lambda}}$.

- **Field** $\Phi(x)$ can be written as:

  $$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ (\nu + h(x) + i\chi(x))/\sqrt{2} \end{pmatrix}$$
The classical Lagrangian.

- In the unitary gauge, the Higgs field has the simple form:

\[ \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h(x) \end{pmatrix} \]
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- The Yukawa Lagrangian has a following form:

\[ \mathcal{L}_{\text{Yukawa}} = -\sum_f m_f \bar{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f h \]
The equation of state.

- The stress-energy density tensor $T_{\mu\nu}$:

$$T_{\mu\nu} = 2 \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} - g_{\mu\nu} \mathcal{L}$$
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- Hamiltonian $H$
  \[ H = \int d^3x T_{00} \]
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- Pressure $P_i$

$$P_i = \int d^3x T_{ii}$$

- We have to calculate average energy density ($\varepsilon$) and average pressure ($P$)
The equation of state.

- The complete Lagrangian has the following form:

\[
\mathcal{L} = -\frac{1}{4} W^{a}_{\mu\nu} W_{\mu\nu}^{\mu, a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum \bar{\psi}_L i\gamma^\mu D_{\mu}\psi_L + \sum \bar{\psi}_R i\gamma^\mu D_{\mu}\psi_R \\
+ (D_{\mu}\Phi)^+ (D^\mu \Phi) - V(\Phi) - \sum f m_f \bar{\psi}_f \psi_f - \sum f \frac{m_f}{v} \bar{\psi}_f \psi_f h
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\[ + (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi) - \sum_f m_f \bar{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f h \]

- For electron and Higgs:

\[ \mathcal{L}_{e,Higgs} = \bar{\psi}_e (i \gamma^\mu \nabla_\mu - m_e) \psi_e + \frac{1}{2} (\partial_\mu h(x))^2 - V(\Phi) \]
The equation of state.

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\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \sum \bar{\psi}_R i \gamma^\mu D_\mu \psi_R \\
+ (D_\mu \Phi)^+ (D^\mu \Phi) - V(\Phi) - \sum_f m_f \bar{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f h
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\]

with

\[
V(\Phi) \sim m_H^2 \left( -\frac{\phi^2}{4} + \frac{\phi^4}{8v^2} \right) + \frac{m_H^2}{4} \left( \frac{\phi^2}{v^2} - 1 \right) h^2(x) + \frac{m_H^2}{8v^2} h^4(x)
\]
The equation of state.

- **Average value** \(< A >\)

\[
\langle A \rangle = \text{Tr}(\rho A), \quad \rho = \frac{1}{Z} e^{-\beta (H - \mu N)} \quad \beta = \frac{1}{K_B T}
\]
The equation of state.

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- **Electron field:**

$$\psi_e = \sum_{\alpha=1}^{2} \int \frac{d^3k}{(2\pi)^3 2E} c(\bar{k},\alpha) u(\bar{k},\alpha) e^{-i k x}$$
The equation of state.

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- The Higgs field:

$$h(x) = \int \frac{d^3k}{(2\pi)^3 2E} [a(\bar{k}) e^{-i k x} + a^+(\bar{k}) e^{i k x}]$$
The equation of state.

- The partition function for fermions (FD):

\[ \langle c^+ c \rangle \sim \frac{1}{e^{\beta(E_k-\mu)} + 1} \]
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- The partition function for bosons (BE):

\[ \langle a^+ a \rangle \sim \frac{1}{e^{\beta (E_k - \mu)} - 1} \]

- The mean field approximation:

\[ \langle A^2 \rangle \sim \langle A \rangle^2 \]
The equation of state.

- Electron part of average energy density:

\[ \varepsilon_e = 8\pi \int_0^\infty d\bar{k} \bar{k}^2 E_k \frac{1}{e^{\beta(E_k - \mu)} + 1} \]
The equation of state.

- Electron part of average energy density:

\[ \varepsilon_e = 8\pi \int_0^\infty \, d\vec{k} \, \vec{k}^2 \, E_k \, \frac{1}{e^{\beta (E_k - \mu)} + 1} \]

- The change of variable \( \vec{k} \rightarrow x \):

\[ \mu = m_e + \bar{\mu}, \quad x = \beta (E_k - m_e), \quad E_k = \sqrt{\vec{k}^2 + m_e^2}, \quad y = \beta \bar{\mu} \]
The equation of state.

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- Fermi-Dirac integrals:

\[ F_j(y) = \int_0^\infty dt \ \frac{t^{\frac{j}{2}}}{e^{(t-y)} \pm 1} \]
The equation of state.

- We can rewrite $\varepsilon_e$

$$
\varepsilon_e = 8\pi (2m_e)^{3/2} (K_B T)^{5/2} \int_{0}^{\infty} dx \frac{x^{3/2} \left( \frac{K_B T x}{2m_e} + 1 \right)^{3/2}}{e^{x-y_e} + 1}
$$

$$
+ 8\pi \sqrt{2} (m_e)^{5/2} (K_B T)^{3/2} \int_{0}^{\infty} dx \frac{x^{1/2} \left( \frac{K_B T x}{2m_e} + 1 \right)^{1/2}}{e^{x-y_e} + 1}
$$
The equation of state.

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\]

\[+ 8\pi \sqrt{2} (m_e)^{5/2} (K_B T)^{3/2} \int_0^\infty dx \frac{x^{1/2} \left( \frac{K_B T x}{2m_e} + 1 \right)^{1/2}}{e^{x-y_e} + 1}\]

- Let’s take very high temperature \( K_B T >> m_e \), then:

\[
\varepsilon_e = 8\pi (K_B T)^4 \int_0^\infty dx \frac{x^3}{e^{x-y_e} + 1} + 8\pi m_e^2 (K_B T)^2 \int_0^\infty dx \frac{x}{e^{x-y_e} + 1}
\]

with

\[
m_e = m_e^{(0)} \frac{\phi}{v}
\]
Similarly, we can calculate $\varepsilon_{\text{Higgs}}$

$$
\varepsilon_{\text{Higgs}} = 2\pi (2m_H)^{3/2}(K_BT)^{5/2} \int_0^\infty dx \frac{x^{3/2} \left( \frac{K_BT x}{2m_H} + 1 \right)^{3/2}}{e^{x-y_H} - 1} 
$$

$$
+ 2\pi \sqrt{2} (m_H)^{5/2}(K_BT)^{3/2} \int_0^\infty dx \frac{x^{1/2} \left( \frac{K_BT x}{2m_H} + 1 \right)^{1/2}}{e^{x-y_H} - 1} 
$$

$$
+ m_H^2 \left( -\frac{\phi^2}{4} + \frac{\phi^4}{8v^2} \right) 
$$

$$
+ \frac{3\pi}{\sqrt{2}} m_H^{5/2} \left( \frac{\phi^2}{v^2} - 1 \right)(K_BT)^{3/2} \int_0^\infty dx \frac{x^{1/2} \left( \frac{K_BT x}{2m_H} + 1 \right)^{1/2}}{e^{x-y_H} - 1} 
$$

$$
+ \frac{m_H^7}{4v^2} (K_BT)^3 \left[ \int_0^\infty dx \frac{x^{1/2} \left( \frac{K_BT x}{2m_H} + 1 \right)^{1/2}}{e^{x-y_H} - 1} \right]^2 
$$

The equation of state.

- Expanding expression in bracket into the series:

\[
\left[ \frac{K_B T x}{2 m_H} + 1 \right]^{\frac{n}{2}} \sim 1 + \frac{n}{2} \frac{K_B T}{2 m_H} x
\]
The equation of state.

- Expanding expression in bracket into the series:

\[
\left[ \frac{K_B T x}{2m_H} + 1 \right]^n \approx 1 + \frac{n}{2} \frac{K_B T}{2m_H} x
\]

- We will get:

\[
\varepsilon_{Higgs} = m_H^2 \left( -\frac{\phi^2}{4} + \frac{\phi^4}{8v^2} \right) + (\ldots)I_H^{(\frac{1}{2})} + (\ldots)I_H^{(\frac{3}{2})} + (\ldots)I_H^{(\frac{5}{2})}
\]

where \( I_H^{(\frac{i}{2})} \) denotes

\[
I_H^{(\frac{i}{2})} = \int_0^\infty dx \frac{x^{\frac{i}{2}}}{e^{x-y_H} - 1}
\]
The equation of state.

- The polylogarithm functions

\[ \text{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} \]
The equation of state.

- The polylogarithm functions

\[ \text{Li}_n(z) = \sum_{k=1}^{\infty} z^k/k^n \]

- Fermi-Dirac integrals have been calculated using

\[ I_{\frac{1}{2}}^{(\frac{1}{2})} = \frac{1}{2} \sqrt{\pi} \text{Li}_{3/2}(e^{y_H}), \quad I_{\frac{3}{2}}^{(\frac{3}{2})} = \frac{3}{4} \sqrt{\pi} \text{Li}_{5/2}(e^{y_H}) \]

\[ I_{\frac{5}{2}}^{(\frac{5}{2})} = \frac{15}{8} \sqrt{\pi} \text{Li}_{7/2}(e^{y_H}), \quad \text{with} \ y_H = -m_H/K_BT \]
The equation of state.

• The polylogarithm functions

\[ \text{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} \]

• Fermi-Dirac integrals have been calculated using MATHEMATICA 5.1

\[ I_{H}^{\left( \frac{1}{2} \right)} = \frac{1}{2} \sqrt{\pi} \text{Li}_{3/2}(e^{y_H}), \quad I_{H}^{\left( \frac{3}{2} \right)} = \frac{3}{4} \sqrt{\pi} \text{Li}_{5/2}(e^{y_H}) \]

\[ I_{H}^{\left( \frac{5}{2} \right)} = \frac{15}{8} \sqrt{\pi} \text{Li}_{7/2}(e^{y_H}), \quad \text{with } y_H = -m_H/K_B T \]

⇒ integrals \[ I_{H}^{\left( \frac{j}{2} \right)} \sim 0 \]
The equation of state.

- Fermi-Dirac integrals for electron part:

\[
I^{(j)}_e = \int_0^\infty dx \frac{x^j}{e^{x-y_e} + 1}
\]
The equation of state.

- Fermi-Dirac integrals for electron part:

\[ I_e^{(j)} = \int_0^\infty dx \frac{x^j}{e^{x-y_e} + 1} \]

\[ I_e^{(1)} = -\text{Li}_2(-e^{y_e}), \quad I_e^{(3)} = -\text{Li}_4(-e^{y_e}) \]
The equation of state.

- Fermi-Dirac integrals for electron part:

\[ I_e^{(j)} = \int_0^\infty dx \frac{x^j}{e^{x-y_e} + 1} \]

\[ I_e^{(1)} = -\text{Li}_2(-e^{y_e}), \quad I_e^{(3)} = -\text{Li}_4(-e^{y_e}) \]

\[ y_e = \left[ \mu - m_e^{(0)} \frac{\Phi}{v} \right] / K_B T \]
The equation of state.

- **Average pressure** $P_i$:

\[ P_i = \frac{8}{3} \pi (2m_e)^{3/2} (K_B T)^{5/2} \int_0^\infty dx \frac{x^{3/2} (\frac{K_B T x}{2m_e} + 1)^{3/2}}{e^{x-y_e} + 1} \]

\[ + \frac{2}{3} \pi (2m_H)^{3/2} (K_B T)^{5/2} \int_0^\infty dx \frac{x^{3/2} (\frac{K_B T x}{2m_H} + 1)^{3/2}}{e^{x-y_H} - 1} \]

\[ - \frac{1}{3} m_H^2 (- \frac{\phi^2}{4} + \frac{\phi^4}{8v^2}) \]

\[ - \frac{\pi}{\sqrt{2}} m_H^{5/2} (\frac{\phi^2}{v^2} - 1) (K_B T)^{3/2} \int_0^\infty dx \frac{x^{1/2} (\frac{K_B T x}{2m_H} + 1)^{1/2}}{e^{x-y_H} - 1} \]

\[ - \frac{m_H^7}{12v^2} (K_B T)^3 \left[ \int_0^\infty dx \frac{x^{1/2} (\frac{K_B T x}{2m_H} + 1)^{1/2}}{e^{x-y_H} - 1} \right]^2 \]
Energy density $\varepsilon$ as a function of $\phi$ for $m_H = 115$ GeV, $v = 200$ GeV, $\mu_e = 1$ GeV and $K_B T = 0$ GeV (left) and $K_B T = 20$ GeV (right).
Results.

Energy density $\varepsilon$ as a function of $\phi$ for $m_H = 115$ GeV, $v = 200$ GeV, $\mu_e = 1$ GeV and $K_B T = 0$ GeV and $K_B T > 0$ GeV.
Equation of state $P/\varepsilon$. 